§111.xx. Algebra I, Beginning with School Year 2013-2014 (One Credit).

(a)   General requirements. This course is recommended for students in Grades 8 or 9. Prerequisite: Mathematics, Grade 8 or its equivalent.

(b)   Introduction.

     (1)   The desire to achieve educational excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the College and Career Readiness Standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

     (2)   The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

     (3)   In Algebra I, students will build on the Texas Essential Knowledge and Skills (TEKS) for Grades 6-8 Mathematics, which provide a foundation in linear relationships, number and operations, and proportionality. Students will study linear, quadratic, and exponential functions and their related transformations, equations, and associated solutions. Students will connect functions and their associated solutions in both mathematical and real-world situations. Students will use technology to collect and explore data and analyze statistical relationships. In addition, students will study polynomials of degree one and two, radical expressions, sequences, and laws of exponents. Students will generate and solve linear systems with two equations and two variables and will create new functions through transformations

     (4)   Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c)   Knowledge and skills.

(1)   Mathematical Process Standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

   (A)   apply mathematics to problems arising in everyday life, society, and the workplace;

   (B)   use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

   (C)   select tools, including real objects, manipulatives, paper/pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

   (D)   communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

   (E)   create and use representations to organize, record, and communicate mathematical ideas;

   (F)   analyze mathematical relationships to connect and communicate mathematical ideas; and

   (G)   display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2)   Linear Functions, Equations, and Inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

   (A)   determine the domain and range of a linear function in mathematical problems and determine reasonable domain and range values for real-world situations, both continuous and discrete;

   (B)   write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y – y_1 = m(x – x_1)$, given one point and the slope and given two points;

   (C)   write linear equations in two variables given a table of values, a graph, and a verbal description;

   (D)   write and solve equations involving direct variation;

   (E)   write linear equations in two variables that contain a given point and are parallel to a given line;

   (F)   write linear equations in two variables that contain a given point and are perpendicular to a given line;

   (G)   write linear equations in two variables that are parallel and lines that are perpendicular to the x- and to the y-axis, and determine whether their slopes are zero or undefined;

(H)   write linear inequalities in two variables given a table of values, a graph, and a verbal description; and

(I)   write systems of two linear equations given a table of values, a graph, and a verbal description.

(3)   Linear Functions, Equations, and Inequalities. The student applies the mathematical process standards when using graphs of linear functions, their key features, and their related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(A)   determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m (x - x_1)$;

(B)   calculate the rate of change of a linear function represented tabularly, graphically, and algebraically over a specified interval within mathematical and real-world problems;

(C)   graph linear functions on the coordinate plane and identify key features including x-intercept, y-intercept, zeros, and slope in mathematical and real-world problems;

(D)   graph the solution set of linear inequalities in two variables on the coordinate plane;

(E)   determine the effects on the graph of the parent function $f (x) = x$ when $f (x)$ is replaced by $a \cdot f (x), f (x) + d , f(x - c), f(b \cdot x)$ for specific values of a, b, c and d;

(F)   graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist;

(G)   estimate graphically the solutions to systems of two linear equations with two variables in real-world problems; and

(H)   graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.

(4)   Linear Functions, Equations, and Inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:

(A)   calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association;

(B)   compare and contrast association and causation in real-world problems; and

(C)   write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

(5)	Linear Functions, Equations, and Inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

(A)	solve linear equations in one variable, including those for which the application of the distributive property is necessary and includes variables on both sides;

(B)	solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and includes variables on both sides; and

(C)	solve, using substitution and Gaussian elimination, systems of two linear equations with two variables for mathematical and real-world problems.

(6)	Quadratic Functions, and Equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:

(A)	determine the domain and range of quadratic functions;

(B)	write equations of quadratic functions given the vertex and another point on the graph, write this equation in vertex form ($f(x) = a(x - h)^2 + k$), and then rewrite this equation from vertex form to standard form ($f(x) = ax^2 + bx + c$); and

(C)	write quadratic functions when given real solutions and graphs of their related equations.

(7)	Quadratic Functions, and Equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:

(A)	graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry;

(B)	describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions; and

(C)	determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $a \cdot f(x)$, $f(x) + d$, $f(x - c)$, $f(K \cdot x)$ for specific values of a, b, c and d.

(8)	Quadratic Functions, and Equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(A) solve quadratic equations, having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula; and

(B) write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

(9) Exponential Functions, and Equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations, and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(A) determine the domain and range of exponential functions of the form $f(x) = a \cdot b^x$

(B) interpret the meaning of the values of a and b in exponential functions of the form $f(x) = a \cdot b^x$ in real-world problems;

(C) write exponential functions in the form $f(x) = a \cdot b^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations including growth and decay;

(D) graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptotes, in mathematical and real-world problems; and

(E) write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.

(10) Number and Algebraic Methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms, and perform operations on, polynomial expressions. The student is expected to:

(A) add and subtract polynomials of degree one and degree two;

(B) multiply polynomials of degree one and degree two;

(C) determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two;

(D) rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property, such as rewriting $(4x)(x - 2)$ as $(4x)(x) - (4x)(2)$, and then writing it as $4x^2 - 8x$, or $4x^2 - 8x$ to $(4x)(x) - (4x)(2)$, and then factoring the result as $(4x)(x - 2)$;

(E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c,$ including perfect square trinomials of degree two; and

(F) decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite it, such

as-rewriting the expression $49x^4 - y^4$ to $(7x^2)^2 - (y^2)^2$, and then factoring it as $(7x^2 + y^2)(7x^2 - y^2)$.

(11)   Number and Algebraic Methods. The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:

    (A)   simplify numerical radical expressions involving square roots; and

    (B)   simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

(12)   Number and Algebraic Methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations and functions. The student is expected to:

    (A)   decide whether relations represented verbally, tabularly, graphically, and symbolically define a function;

    (B)   evaluate functions, expressed in function notation, given one or more elements in their domains;

    (C)   identify terms of arithmetic and geometric sequences when the sequences are given in function form and given in recursive form;

    (D)   write a formula for the nth term of arithmetic and geometric sequences, given the value of several of their terms; and

    (E)   solve mathematic and scientific formulas, and other literal equations, for a specified variable.

§111.xx. Algebra II, Beginning with School Year 2013-2014 (One-half to One Credit).

(a)   General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisite: Algebra I.

(b)   Introduction.

   (1)   The desire to achieve educational excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the College and Career Readiness Standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

   (2)   The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

   (3)   In Algebra II students build on the foundations from Kindergarten to Grade 8 and Algebra I. Students broaden their knowledge of quadratic functions, exponential functions, and systems of equations. They study logarithmic, square root, cubic, cube root, absolute value, rational functions, and their related equations. Students connect functions to their inverses and to their associated equations and solutions in both mathematical and real-world situations. In addition, students extend their knowledge of data analysis and numeric and algebraic methods.

   (4)   Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c)   Knowledge and skills.

   (1)   Mathematical Process Standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

      (A)   apply mathematics to problems arising in everyday life, society, and the workplace;

(B)   use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C)   select tools, including real objects, manipulatives, paper/pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D)   communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E)   create and use representations to organize, record, and communicate mathematical ideas;

(F)   analyze mathematical relationships to connect and communicate mathematical ideas; and

(G)   display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2)   Attributes of Functions and their Inverses. The student applies mathematical processes to understand that functions have distinct key attributes and to understand the relationship between a function and its inverse. The student is expected to:

(A)   graph the functions, $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x)=\sqrt[3]{x}$, $f(x) = b^x$, and $f(x) = log_b(x)$ where b is 2, 10 and e, and when applicable analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and relative maxima and minima given an interval;

(B)   graph and write the inverse of a function;

(C)   describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restrictions on domains and ranges; and

(D)   use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.

(3)   Systems of Equations and Inequalities. The student applies mathematical processes to formulate systems of equations and inequalities, to use a variety of methods to solve, and to analyze reasonableness of solutions. The student is expected to:

(A)   formulate systems of equations, including systems consisting of three linear equations in three variables and systems consisting of two equations, the first linear and the second quadratic.

(B)   solve systems of three linear equations in three variables by methods such as elimination, using technology with matrices, and substitution;

(C)   solve, algebraically, systems of two equations in two variables consisting of a linear equation and a quadratic equation;

(D)   determine the reasonableness of solutions to systems of a linear equation and a quadratic equation in two variables;

(E)   formulate systems of at least two linear inequalities in two variables;

(F)   solve systems of two or more linear inequalities in two variables; and

(G)   determine possible solutions in the solution set of systems of two or more linear inequalities in two variables.

(4)   Quadratic and Square Root Functions, Equations, and Inequalities. The student applies mathematical processes to understand that quadratic and square root functions and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

(A)   write the quadratic function given three specified points in the plane;

(B)   write the equation of a parabola using given attributes, including vertex, focus, directrix, axis of symmetry, and direction of opening;

(C)   determine the effect on the graph of $f(x) = \sqrt{x}$ when $f(x)$ is replaced by a $f(x), f(x) + d, f(bx)$, and $f(x\text{-}c)$ for specific positive and negative values of $a, b, c,$ and $d;$

(D)   transform a quadratic function $f(x) = ax^2 + bx + c$ to the form $f(x) = a(x-h)^2 + k$ to identify the different attributes of $f(x)$;

(E)   formulate quadratic and square root equations;

(F)   solve quadratic and square root equations;

(G)   identify extraneous solutions of square root equations ; and

(H)   solve quadratic inequalities.

(5)   Exponential and Logarithmic Functions and Equations. The student applies mathematical processes to understand that exponential and logarithmic functions can be used to model situations and solve problems. The student is expected to:

(A)   determine the effects on the key attributes on the graphs of $f(x) = b^x$ and $f(x) = log_b(x)$ where b is 2, 10 and e when $f(x)$ is replaced by $a f(x), f(x) + d,$ and $f(x\text{-}c)$ for specific positive and negative values of a, c and $d;$

(B)   formulate exponential and logarithmic equations that model real-world situations;

(C)   rewrite exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations;

(D)   solve exponential equations of the form $y = a{\cdot}b^x$ where a is a nonzero real number and b is greater than zero and not equal to one and single logarithmic equations that have real solutions; and

(E)   determine the reasonableness of a solution to a logarithmic equation.

(6)   Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities. The student applies mathematical processes to understand that cubic, cube root, rational, and absolute value functions and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

    (A)   analyze the effect on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ when $f(x)$ is replaced by $a{\cdot}f(x), f(bx), f(x{-}c)$, and $f(x) + d$ for specific positive and negative values of $a, b, c,$ and d;

    (B)   solve cube root equations;

    (C)   analyze the effect on the graphs of $f(x) = |x|$ when $f(x)$ is replaced by $a{\cdot}f(x), f(bx), f(x{-}c)$, and $f(x) + d$ for specific positive and negative values of a, b, c and d;

    (D)   formulate absolute value linear equations that model real-world situations;

    (E)   solve absolute value linear equations;

    (F)   solve absolute value linear inequalities;

    (G)   analyze the effect on the graphs of $f(x) = \frac{1}{x}$, when $f(x)$ is replaced by $a\,f(x), f(bx), f(x{-}c)$, and $f(x) + d$ for specific positive and negative values of $a, b, c,$ and d;

    (H)   formulate rational equations that model real-world situations;

    (I)   solve rational equations that have real solutions;

    (J)   determine the reasonableness of a solution to a rational equation;

    (K)   determine the restrictions on the domain of a rational function; and

    (L)   formulate and solve equations involving inverse variation.

(7)   Number and Algebraic Methods. The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to:

    (A)   add, subtract, and multiply complex numbers;

    (B)   add, subtract, and multiply polynomials;

    (C)   determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two;

    (D)   determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods such as the Remainder Theorem;

    (E)   determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping;

(F)   determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and of degree two;

(G)   rewrite radical expressions that contain variables to equivalent forms;

(H)   solve equations involving rational exponents; and

(I)   write the domain and range of a function in interval notation.

(8)   Data. The student applies mathematical processes to analyze data, select appropriate models, write corresponding functions, and make predictions. The student is expected to:

(A)   analyze data to select the appropriate model from among linear, quadratic, and exponential models;

(B)   use regression methods available through technology to write a linear function, a quadratic function, and an exponential function from a given set of data; and

(C)   predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models.

§111.xx. Geometry, Beginning with School Year 2013-2014 (One Credit).

(a)   General requirements. Students shall be awarded one credit for successful completion of this course. Prerequisite: Algebra I.

(b)   Introduction.

   (1)   The desire to achieve educational excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the College and Career Readiness Standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

   (2)   The process standards integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

   (3)   In Geometry, students build on the foundations from K-8 and Algebra I to strengthen their mathematical reasoning skills in geometric contexts. Within the course, students will begin to focus on more precise terminology, symbolic representations, and the development of proofs. Students will explore concepts covering coordinate and transformational geometry; logical argument and constructions; proof and congruence; similarity, proof, and trigonometry; two- and three- dimensional figures; circles; and probability. Students will connect previous knowledge from Algebra I to Geometry through the coordinate and transformational geometry strand. In the logical arguments and constructions strand, students are expected to create formal constructions using a straight edge and compass. Though this course is primarily Euclidean geometry, students should complete the course with an understanding that non-Euclidean geometries exist. In proof and congruence, students will use deductive reasoning to justify, prove and apply theorems about geometric figures. Throughout the standards, to "prove" means a formal proof to be shown in a paragraph, flow chart, or two-column formats. Proportionality is the unifying component of the similarity, proof and trigonometry strand and students will use their proportional reasoning skills to prove and apply theorems and solve problems in this strand. The two- and three-dimensional figure strand focuses on the application of formulas in

multi-step situations because students have developed their background knowledge in two-and three-dimensional figures. Using patterns to identify geometric properties, students will apply theorems about circles to determine relationships between special segments and angles in circles. Due to the emphasis of probability and statistics in the College and Career Readiness Standards, standards dealing with probability have been added to the Geometry curriculum to ensure students have proper exposure to these topics before pursuing their post-secondary education.

(4)   These standards are meant to provide clarity and specificity in regards to the content covered in the high school Geometry course. These standards are not meant to limit the methodologies utilized to convey this knowledge to students. Though the standards are written in a particular order, they are not necessarily meant to be taught in the given order. In the standards, the phrase "to solve problems" includes both contextual and non-contextual problems unless specifically stated.

(5)   Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c)   Knowledge and skills.

(1)   Mathematical Process Standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A)   apply mathematics to problems arising in everyday life, society, and the workplace;

(B)   use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C)   select tools, including real objects, manipulatives, paper/pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D)   communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E)   create and use representations to organize, record, and communicate mathematical ideas;

(F)   analyze mathematical relationships to connect and communicate mathematical ideas; and

(G)   display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2)   Coordinate and Transformational Geometry. The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:

   (A)   determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint;

   (B)   derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines; and

   (C)   determine an equation of a line parallel or perpendicular to a given line that passes through a given point.

(3)   Coordinate and Transformational Geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

   (A)   describe and perform transformations of figures in a plane using coordinate notation, i.e. $(x, y) \rightarrow (-x, y)$;

   (B)   determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane;

   (C)   identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane; and

   (D)   identify and distinguish between reflectional and rotational symmetry in a plane figure.

(4)   Logical Argument and Constructions. The student uses the process skills with inductive reasoning to understand geometric relationships. The student is expected to:

   (A)   distinguish between undefined terms, definitions, postulates, conjectures, and theorems;

   (B)   identify and determine the validity of the converse, inverse, and contrapositive of a conditional statement and recognize the connection between a biconditional statement and a true conditional statement with a true converse;

   (C)   verify that a conjecture is false using a counterexample; and

(D) compare geometric relationships between Euclidean and spherical geometries, including parallel lines and the sum of the angles in a triangle.

(5) Logical Argument-and Constructions. The student uses constructions to validate conjectures about geometric figures. The student is expected to:

(A) investigate patterns to make conjectures about geometric relationships, including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles choosing from a variety of tools such as compass and straightedge, paper folding, and dynamic geometric software;

(B) construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge;

(C) use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships; and

(D) verify the Triangle Inequality theorem using constructions and apply the theorem to solve problems.

(6) Proof and Congruence. The student uses the process skills with deductive reasoning to prove and apply theorems by utilizing a variety of methods (coordinate, transformational, axiomatic) and formats (two-column, paragraph, flow chart). The student is expected to:

(A) prove theorems about angles formed by the intersection of lines and line segments, including vertical angles, angles formed by parallel lines cut by a transversal, and equidistance between the endpoints of a segment and points on its perpendicular bisector, and apply these relationships to solve problems;

(B) prove two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle,-Side-Side-Side, Angle-Angle-Side, and Hypotenuse-Leg congruence conditions;

(C) apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles;

(D) prove theorems about the relationships in triangles, including the sum of interior angles, base angles of isosceles triangles, midsegments, and medians and apply these relationships to solve problems; and

(E)   prove a quadrilateral is a parallelogram, rectangle, square or rhombus using opposite sides, opposite angles, or diagonals and apply these relationships to solve problems.

(7)   Similarity, Proof, and Trigonometry. The student uses the process skills in applying similarity to solve problems. The student is expected to:

(A)   apply the definition of similarity in terms of a dilation to identify similar figures and their proportional sides and the congruent corresponding angles; and

(B)   apply the Angle-Angle criterion to verify similar triangles and apply the proportionality of the corresponding sides to solve problems.

(8)   Similarity, Proof, and Trigonometry. The student uses the process skills with deductive reasoning to prove and apply theorems by utilizing a variety of methods (coordinate, transformational, axiomatic) and formats (two-column, paragraph, flow chart). The student is expected to:

(A)   prove theorems about similar triangles, including the Triangle Proportionality theorem, and apply these theorems to solve problems; and

(B)   identify and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle, including the geometric mean, to solve problems.

(9)   Similarity, Proof, and Trigonometry. The student uses the process skills to understand and apply relationships in right triangles. The student is expected to:

(A)   determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems;

(B)   apply the relationships in special right triangles ($30° - 60° - 90°$ and $45° - 45° - 90°$) and the Pythagorean theorem, including Pythagorean triples, to solve problems.

(10)   Two-dimensional and three-dimensional figures. The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to:

(A)   identify the shapes of two-dimensional cross-sections of prisms, pyramids, cylinders, cones, and spheres and identify three-dimensional objects generated by rotations of two-dimensional shapes; and

(B)   determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change.

(11)   Two-dimensional and three-dimensional figures. The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to:

(A)   apply the formula for the area of regular polygons to solve problems using appropriate units of measure;

(B)   determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure;

(C)   apply the formulas for the total and lateral surface area of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure; and

(D)   apply the formulas for the volume of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure.

(12)   Circles. The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to:

(A)   apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems;

(B)   apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems;

(C)   apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve problems;

(D)   describe radian measure of an angle as the ratio of the length of an arc intercepted by a central angle and the radius of the circle; and

(E)   show that the equation of a circle with center at the origin and radius r is $x^2 + y^2 = r^2$ and determine the equation for the graph of a circle with radius r and center (h, k), $(x - h)^2 + (y - k)^2 = r^2$.

(13)   Probability. The student uses the process skills to understand probability in real-world situations and how to apply independence and dependence of events. The student is expected to:

(A)   develop strategies to use permutations and combinations to solve contextual problems;

(B)   determine probabilities based on area to solve contextual problems;

(C)   identify whether two events are independent and compute the probability of the two events occurring together with or without replacement;

(D)   apply conditional probability in contextual problems; and

(E)   apply independence in contextual problems.

DRAFT

§111.xx. Precalculus, Beginning with School Year 2013-2014 (One-half to One Credit).

(a)   General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: two years of algebra and one year of geometry.

(b)   Introduction.

   (1)   The desire to achieve educational excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the College and Career Readiness Standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

   (2)   The process standards integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

   (3)   Precalculus is the preparation for calculus. The course approaches topics from a function point of view, where appropriate, and is designed to strengthen and enhance conceptual understanding and mathematical reasoning used when modeling and solving mathematical and real-world problems. Students systematically work with functions and their multiple representations. The study of Precalculus deepens students' mathematical understanding and fluency with algebra and trigonometry and extends their ability to make connections and apply concepts and procedures at higher levels. Students investigate and explore mathematical ideas, develop multiple strategies for analyzing complex situations, and use technology to build understanding, make connections between representations, and provide support in solving problems.

   (4)   Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c)   Knowledge and skills.

   (1)   Mathematical Process Standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A)   apply mathematics to problems arising in everyday life, society, and the workplace;

(B)   use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C)   select tools, including real objects, manipulatives, paper/pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D)   communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E)   create and use representations to organize, record, and communicate mathematical ideas;

(F)   analyze mathematical relationships to connect and communicate mathematical ideas; and

(G)   display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

(2)   Functions. The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

(A)   use the composition of two functions to model and solve real-world problems;

(B)   demonstrate that function composition is not always commutative;

(C)   represent a given function as a composite function of two or more functions;

(D)   describe symmetry of graphs of even and odd functions;

(E)   determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations;

(F)   graph exponential logarithmic rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions;

(G)   graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power functions and their transformations, including $a \cdot f(x), f(x) + d, f(x - c), f(b \cdot x)$ for specific values of a, b, c, and d, in mathematical and real-world problems;

(H)   graph arcsin x and arccos x and describe the limitations on the domain;

(I)   determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing;

(J)   analyze and describe end behavior of functions, including exponential, logarithmic, rational, polynomial, and power functions using infinity notation to communicate this characteristic in mathematical and real-world problems;

(K)   analyze characteristics of rational functions and the behavior of the function around the asymptotes, including horizontal, vertical, and oblique asymptotes;

(L)   determine various types of discontinuities in the interval ($-\infty$, ∞) as they relate to functions such as rational and piecewise defined functions and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities;

(M)   describe the left-sided behavior and the right-sided behavior of the graph of a function around discontinuities;

(N)   analyze situations modeled by functions, including exponential, logarithmic, rational, polynomial, and power functions, to solve real-world problems such as problems involving growth and decay and optimization;

(O)   develop and use a sinusoidal function that models a situation in mathematical and real-world problems; and

(P)   determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.

(3)   Relations and Geometric Reasoning. The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

(A)   graph a set of parametric equations;

(B)   convert parametric equations into rectangular relations and convert rectangular relations into parametric equations;

(C)   use parametric equations to model and solve mathematical and real-world problems;

(D)   graph points in the polar coordinate system and convert between rectangular coordinates and polar coordinates;

(E)   graph polar equations such as cardiods, limaçons, or lemniscates by plotting points and using technology;

(F)   determine the conic section formed when a plane intersects a double-napped cone;

(G)   make connections between the locus definition of conic sections and their equations in rectangular coordinates;

(H)   use the characteristics of an ellipse to write the equation of an ellipse with center (h, k); and

(I)   use the characteristics of a hyperbola to write the equation of a hyperbola with center (h, k).

(4)   Number and Measure. The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

(A)   determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems;

(B)   describe the relationship between degree and radian measure on the unit circle;

(C)   represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position;

(D)   represent angles in radians or degrees based on the concept of rotation in mathematical and real-world problems, including linear and angular velocity;

(E)   determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems;

(F)   use trigonometry in mathematical and real-world problems, including directional bearing;

(G)   use the Law of Sines in mathematical and real-world problems;

(H)   use the Law of Cosines in mathematical and real-world problems;

(I)   use vectors to model situations involving magnitude and direction;

(J)   represent the addition of vectors and the multiplication of a vector by a scalar geometrically and symbolically; and

(K)   apply vector addition and multiplication of a vector by a scalar in mathematical and real-world problems.

(5)   Algebraic Reasoning. The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

(A)   represent finite sums and infinite series using sigma notation;

(B)   evaluate finite sums and geometric series when possible written in sigma notation;

(C)   represent arithmetic sequences and geometric sequences using recursive formulas;

(D) calculate the *n*th term and the *n*th partial sum of an arithmetic series in mathematical and real-world problems;

(E) represent arithmetic series and geometric series using sigma notation;

(F) calculate the *n*th term of a geometric series, the nth partial sum of a geometric series, and sum of an infinite geometric series when it exists;

(G) apply the Binomial Theorem for the expansion of $(a + b)^n$ in powers of *a* and *b* for a positive integer *n*, where *a* and *b* are any numbers;

(H) use the properties of logarithms to evaluate or transform logarithmic expressions;

(I) generate and solve logarithmic equations in mathematical and real-world problems;

(J) generate and solve exponential equations in mathematical and real-world problems;

(K) solve polynomial equations with real coefficients by applying a variety of techniques such as factoring, graphical methods, or technology in mathematical and real-world problems;

(L) solve polynomial inequalities with real coefficients by applying a variety of techniques such as factoring, graphical methods, or technology and write the solution set of the polynomial inequality in interval notation in mathematical and real-world problems;

(M) solve rational inequalities with real coefficients by applying a variety of techniques such as factoring, graphical methods, or technology and write the solution set of the rational inequality in interval notation in mathematical and real-world problems;

(N) use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions; and

(O) generate and solve trigonometric equations in mathematical and real-world problems.

§111.xx. Mathematical Models with Applications, Beginning with School Year 2013-2014 (One-half to One Credit).

(a)   General requirements. Students can be awarded one-half to one credit for successful completion of this course. Prerequisite: Algebra I. This course must be taken before receiving credit for Algebra II.

(b)   Introduction.

(1)   The desire to achieve educational excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the College and Career Readiness Standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2)   The process standards integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3)   Mathematical Models with Applications is designed to build on knowledge and skills from Kindergarten to Grade 8 and Algebra I. This math course provides a path for students to succeed in Algebra II and prepares them for various post-secondary choices. Students learn to apply mathematics through experiences in personal finance, science, engineering, fine arts, and social science. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure; model information; solve problems; and communicate solutions. Students will select from tools such as physical objects, manipulatives, technology (including graphing calculators, data collection devices, and computers), paper/pencil and from methods such as algebraic techniques, geometric reasoning, patterns, and mental math to solve problems.

(4)   In this course students will use a mathematical modeling cycle to analyze problems, understand problems better, and improve decisions. A basic mathematical modeling cycle is summarized below. The student will:

(A) Represent:

    (i) identify the variables in the problem and select those that represent essential features;

    (ii) formulate a model by creating and selecting from representations such as geometric, graphical, tabular, algebraic, or statistical that describe the relationships between the variables;

(B) Compute: analyze and perform operations on these relationships to draw conclusions;

(C) Interpret: interpret the results of the mathematics in terms of the original problem;

(D) Revise: confirm the conclusions by comparing the conclusions with the problem and revising as necessary; and

(E) Report: report on the conclusions and the reasoning behind them.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical Process Standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper/pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2)   Mathematical Modeling in Personal Finance. The student uses mathematical processes with graphical and numerical techniques to study patterns and analyze data related to personal finance. The student is expected to:

       (A)   use rates and linear functions to solve problems involving personal finance and budgeting, including compensations and deductions;

       (B)   solve problems involving personal taxes; and

       (C)   analyze data to make decisions about banking, including options for online banking, checking accounts, overdraft protection, processing fees, and debit card/ATM fees;

(3)   Mathematical Modeling in Personal Finance. The student uses mathematical processes with algebraic formulas, graphs, and amortization modeling to solve problems involving credit. The student is expected to:

       (A)   use formulas to generate tables to display series of payments for loan amortizations resulting from financed purchases;

       (B)   analyze personal credit options in retail purchasing and compare relative advantages and disadvantages of each option;

       (C)   use technology to create amortization models to investigate home financing and compare buying a home to renting a home; and

       (D)   use technology to create amortization models to investigate automobile financing and compare buying a vehicle to leasing a vehicle.

(4)   Mathematical Modeling in Personal Finance. The student uses mathematical processes with algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. The student is expected to:

       (A)   analyze and compare coverage options and rates in insurance;

       (B)   investigate and compare investment options, including stocks, bonds, annuities, certificates of deposit, and retirement plans; and

       (C)   analyze types of savings options involving simple and compound interest and compare relative advantages of these options.

(5)   Mathematical Modeling in Science and Engineering. The student applies mathematical processes with algebraic techniques to study patterns and analyze data as it applies to science. The student is expected to:

       (A)   use proportionality and inverse variation to describe physical laws such as Hook's Law, Newton's Second Law of Motion, and Boyle's Law;

       (B)   use exponential models available through technology to model growth and decay in areas such as population, biology, ecology, and chemistry, including radioactive decay; and

(C) use quadratic functions to model motion such as an object dropped, bounced, thrown, or kicked.

(6) Mathematical Modeling in Science and Engineering. The student applies mathematical processes with algebra and geometry to study patterns and analyze data as it applies to architecture and engineering. The student is expected to:

(A) use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in architecture;

(B) use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields such as engineering drawing, architecture, and construction;

(C) use the Pythagorean Theorem and special right-triangle relationships to calculate distances; and

(D) use trigonometric ratios to calculate distances and angle measures as applied to fields such as surveying, navigation, and orienteering.

(7) Mathematical Modeling in Fine Arts. The student uses mathematical processes with algebra and geometry to study patterns and analyze data as it applies to fine arts. The student is expected to:

(A) use trigonometric ratios and functions available through technology to model periodic behavior in art and music;

(B) use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and photography;

(C) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music; and

(D) use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields such as painting, sculpture, and photography.

(8) Mathematical Modeling in Social Sciences. The student applies mathematical processes to determine the number of elements in a finite sample space and compute the probability of an event. The student is expected to:

(A) determine the number of ways an event such as a sports tournament may occur using combinations, permutations, and the Fundamental Counting Principle;

(B) compare theoretical to empirical probability such as determining if a particular game of chance is fair; and

(C)   use experiments to determine the reasonableness of a theoretical model such as binomial or geometric.

(8)   Mathematical Modeling in Social Sciences. The student applies mathematical processes and mathematical models to analyze data as it applies to social sciences. The student is expected to:

    (A)   interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, dot plots, stem-and-leaf plots, and box and whisker plots to draw conclusions from the data and determine the strengths and weaknesses of conclusions;

    (B)   analyze numerical data using measures of central tendency (mean, median, and mode) and variability (range, interquartile range or IQR, and standard deviation) in order to make inferences with normal distributions;

    (C)   distinguish the purposes and differences among types of research, including surveys, experiments, and observational studies;

    (D)   use data from a sample to estimate population mean or population proportion;

    (E)   analyze marketing claims based on graphs and statistics from electronic and print media and justify the validity of stated or implied conclusions; and

    (F)   use regression methods available through technology to model linear and exponential functions, interpret correlations, and make predictions.

(9)   Mathematical Modeling in Social Sciences. The student applies mathematical processes to design a study and use graphical, numerical, and analytical techniques to communicate the results of the study. The student is expected to:

    (A)   formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions; and

    (B)   communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project through the use of one or more of a written report, a visual display, an oral report, or a multi-media presentation.

§111.xx. Advanced Quantitative Reasoning, Beginning with School Year 2013-2014 (One-half to One Credit).

(a)   General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Geometry and Algebra II.

(b)   Introduction.

   (1)   The desire to achieve educational excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the College and Career Readiness Standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

   (2)   The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

   (3)   In Advanced Quantitative Reasoning, students develop and apply skills necessary for college, careers, and life. Course content consists primarily of applications of high school mathematics concepts to prepare students to become well-educated and highly informed 21st century citizens. The student develops and applies reasoning, planning, and communication to make decisions and solve problems in applied situations involving numerical reasoning, probability, statistical analysis, finance, mathematical selection, and modeling with algebra, geometry, trigonometry, and discrete mathematics.

   (4)   Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c)   Knowledge and skills.

   (1)   Mathematical Process Standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

      (A)   apply mathematics to problems arising in everyday life, society, and the workplace;

(B)   use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C)   select tools, including real objects, manipulatives, paper/pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D)   communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E)   create and use representations to organize, record, and communicate mathematical ideas;

(F)   analyze mathematical relationships to connect and communicate mathematical ideas; and

(G)   display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2)   Numeric Reasoning. The student applies the process standards in mathematics to generate new understandings by extending existing knowledge. The student generates new mathematical understandings through problems involving numerical data that arise in everyday life, society, and the workplace. The student extends existing knowledge and skills to analyze real-world situations. The student is expected to:

(A)   compare and contrast precision and accuracy in real-life situations such as in measurements and significant figures;

(B)   apply and analyze published ratings, weighted averages, and indices to make informed decisions;

(C)   solve problems involving quantities that are not easily measured using proportionality such as packing problems, crowd estimation, and white blood cell count;

(D)   solve geometric problems involving indirect measurement, including similar triangles, the Pythagorean Theorem, Law of Sines, Law of Cosines, and the use of dynamic geometry software;

(E)   solve problems involving large quantities using combinatorics such as numbers of unique license plates and telephone numbers;

(F)   use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems, including geometric transformations;

(G)   analyze various voting and selection processes to compare results in given situations such as at-large versus single-member districts and plurality versus majority voting; and

(H) select and apply an algorithm of interest to solve real-life problems such as problems using recursion or iteration involving population growth or decline, fractals, and compound interest; the validity in recorded and transmitted data using checksums and hashing; sports rankings, weighted class rankings, and search engine rankings; and problems involving scheduling or routing situations using vertex-edge graphs, critical paths, Euler paths, and minimal spanning trees; and communicate to peers the application of the algorithm in precise mathematical and nontechnical language.

(3) Algebraic Reasoning (Expressions, Equations, and Generalized Relationships). The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve dynamic real-world problems. The student is expected to:

(A) collect numerical bivariate data to create a scatterplot, select a function to model the data, justify the model selection, and use the model to interpret results and make predictions;

(B) describe the degree to which uncorrelated variables may or may not be related and analyze situations where correlated variables do or do not indicate a cause and effect relationship;

(C) determine or analyze an appropriate growth or decay model for problem situations, including linear, exponential, and logistic functions;

(D) determine or analyze an appropriate cyclical model for problem situations that can be modeled with periodic functions;

(E) determine or analyze an appropriate piecewise model for problem situations;

(F) create, represent, and analyze mathematical models for various types of income calculations to determine the best option for a given situation;

(G) create, represent, and analyze mathematical models for expenditures, including those involving credit, to determine the best option for a given situation; and

(H) create, represent, and analyze mathematical models and appropriate representations, including formulas and amortization tables, for various types of loans and investments to determine the best option for a given situation such as cell phone plans and buying versus leasing a car.

(4) Probabilistic and Statistical Reasoning. The student utilizes the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-

solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to effectively communicate the results of student-generated statistical studies and the critical analysis of published statistical studies. The student is expected to:

(A)   use a two-way frequency table as a sample space to identify whether two events are independent and to interpret the results;

(B)   use the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, in mathematical and real-world problems;

(C)   calculate conditional probabilities and probabilities of compound events using tree diagrams, Venn diagrams, area models, and formulas such as Bayes' Theorem;

(D)   interpret conditional probabilities and probabilities of compound events by analyzing representations to make decisions in problem situations;

(E)   use probabilities to make and justify decisions about risks in everyday life such as the lottery, weather forecasts, and insurance costs;

(F)   calculate expected value to analyze mathematical fairness, payoff, and risk;

(G)   determine the validity of logical arguments that include compound conditional statements by constructing truth tables;

(H)   identify limitations and lack of relevant information in studies reporting statistical information especially when studies are reported in condensed form;

(I)   interpret and compare statistical results using appropriate technology given a margin of error in situations such as polls, quality control, and measurements;

(J)   identify potential misuses of statistics to justify particular conclusions, including assertions of a cause-and-effect relationship rather than an association, and missteps or fallacies in logical reasoning such as confounding variables and hasty generalizations;

(K)   describe strengths and weaknesses of sampling techniques, data and graphical displays, and interpretations of summary statistics and other results appearing in a study, including reports published in the media;

(L)   determine the need for and purpose of a statistical investigation and what type of statistical analysis can be used to answer a specific question or set of questions;

(M)   identify the population of interest for a statistical investigation, select an appropriate sampling technique, and collect data;

(N)   identify the variables to be used in a study;

(O)   determine possible sources of statistical bias in a study and how bias may affect the validity of the results;

(P)   create data displays for given data sets to investigate, compare, and estimate center, shape, spread, and unusual features of the data;

(Q)   analyze possible sources of data variability, including those that can be controlled and those that cannot be controlled;

(R)   report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied;

(S)   justify the design and the conclusion(s) of statistical studies, including the methods used; and

(T)   communicate statistical results in oral and written formats using appropriate statistical and nontechnical language.

§111.xx. Independent Study in Mathematics, Beginning with School Year 2013-2014 (One-half to One Credit).

(a)   General requirements.

   (1)   Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Geometry and Algebra II.

   (2)   Students may repeat this course with different course content for up to three credits.

   (3)   The requirements for each course must be approved by the local district before the course begins.

   (4)   If this course is being used to satisfy requirements for the Distinguished Achievement Program, student research/products must be presented before a panel of professionals or approved by the student's mentor.

(b)   Introduction.

   (1)   The desire to achieve educational excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the College and Career Readiness Standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

   (2)   The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

   (3)   In Independent Study in Mathematics, students will extend their mathematical understanding beyond the Algebra II level in a specific area or areas of mathematics such as theory of equations, number theory, non-Euclidean geometry, linear algebra, advanced survey of mathematics, or history of mathematics.

   (4)   Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c)   Mathematical Process Standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

    (1)   apply mathematics to problems arising in everyday life, society, and the workplace;

    (2)   use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

    (3)   select tools, including real objects, manipulatives, paper/pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

    (4)   communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

    (5)   create and use representations to organize, record, and communicate mathematical ideas;

    (6)   analyze mathematical relationships to connect and communicate mathematical ideas; and

    (7)   display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communications.